Optimal Advertising and Promotion Budgets in Dynamic Markets with Brand Equity as a Mediating Variable

S. Sriram♣
School of Business
University of Connecticut
Storrs, CT 06269
Ph: (860) 486 9351
Email: ssriram@business.uconn.edu

Manohar U. Kalwani
Krannert Graduate School of Management
Purdue University
West Lafayette, IN 47907
Ph: (765) 494 4400
Email: kalwani@purdue.edu

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Abstract

We study the optimal levels of advertising and promotion budgets in dynamic markets with brand equity as a mediating variable. To this end, we develop and estimate a state-space model based on the Kalman filter that captures the dynamics of brand equity as influenced by its drivers that include a brand's advertising and sales promotion expenditures. By integrating the Kalman filter with the random coefficients logit demand model, our estimation allows us to capture the dynamics of brand equity as well as model consumer heterogeneity using store-level data. Using these demand model estimates, we determine the Markov Perfect Equilibrium advertising and promotion strategies. Our empirical analysis is based on store-level scanner data in the orange juice category, which comprises two major brands – Tropicana and Minute Maid. The calibration of the demand model reveals that sales promotions have a significant positive effect on consumers' utility and induce consumers to switch to the promoted brand. However, there is also a negative effect of promotions on brand equity that carries over from period to period. Overall, we find that while sales promotions have a net positive impact both in the short-term and in the long-term, the implied total elasticity including the long-term effect is smaller than the short-term elasticity. Correspondingly, we expect myopic decision-makers to allocate higher than optimal expenditures to sales promotions. Our results from the supply side analysis reveal that the equilibrium forward-looking promotion levels are higher for Minute Maid, the brand for which the adverse long-term effect of promotions is lower. However, the observed promotion levels are higher for Tropicana compared to Minute Maid, a result consistent only with the myopic case. Further, our results reveal that the actual promotion levels for both brands are higher than the optimal budgets for the forward-looking as well as the two-year planning horizon scenarios. Hence, it may be profitable for both brands to reduce their promotion levels. The equilibrium forward-looking advertising levels are higher for Tropicana, the brand that has a higher responsiveness to advertising. Further, the optimal forward-looking advertising levels are higher than the optimal budgets for the myopic as well as the two-year planning horizon scenarios. However, these levels are higher than the actual advertising expenditures for both brands. Hence, it may be optimal for both brands to reduce their advertising levels.

1. INTRODUCTION

The optimal allocation of resources between the various elements of the marketing mix remains an important issue relevant to both academics and practitioners alike. In fact, the problems surrounding advertising and sales promotion spending continue to attract considerable managerial attention. Given the need to meet short-term sales and market share objectives, brand managers in packaged goods firms are under pressure to increase or maintain high sales promotion spending at the expense of advertising (Low and Mohr 2000). However, with documented evidence of the possible detrimental effects of sales promotions over the long-term (Mela, Gupta, and Lehman 1997; Papatla and Krishnamurthi 1996; Jedidi, Mela, and Gupta 1999), managers in many consumer packaged goods firms are trying to increase their expenditures on brand building activities such as advertising while cutting down their sales and trade promotion budgets. For example, starting in the early 1990s, Procter & Gamble reduced its trade promotion spending and adopted an everyday low pricing strategy (Reitman 1992) while increasing its advertising expenditures (Ailawadi, Lehmann, and Neslin 2001). The example illustrates the types of tradeoffs that managers have to make between the short-term benefits of marketing actions versus their potential adverse long-term consequences. The issue of the optimal allocation of the advertising and sales promotion budgets is especially critical for brand managers who are faced with flat or declining marketing budgets (Low and Mohr 2000).

A key purpose of our study is to investigate the optimal allocation of advertising and promotion budgets when both these marketing instruments have long-term effects. Specifically, we consider the tradeoffs that managers make between the short-term benefits of marketing actions such as sales promotions and their possible detrimental long-term effects. Our goal is to understand how the optimal marketing budgets under short-term orientation differ from those that consider the long-term consequences of marketing actions. To this end, we formulate and estimate a demand model wherein the long-term effects of these variables are modeled with brand equity as the mediating variable. This is in contrast to most of the extant literature (with the exception of Jedidi, Mela, and Gupta 1999) which examines the long-term implications of these marketing actions with sales or market share as the mediating variable (see, for example, Dekimpe and Hanssens 1999; Naik, Raman, and Winer 2005). The use of brand equity as a mediating variable allows us to examine whether there are adverse long-term consequences of, say, sales promotions, even if their impact in the short-term on the brand's sales and profitability is positive. We use the demand model parameter estimates to compare the optimal Markov Perfect Equilibrium advertising and promotion budgets under the scenarios when the decision makers consider only the short-term consequences of marketing programs versus when they are long-term oriented.

Our demand side model is similar in spirit to Jedidi, Mela, and Gupta (1999) in that we model the long-term (potentially negative) effects of sales promotions and advertising with brand equity as the mediating variable. However, we present an alternative way of modeling dynamic parameters using the Kalman filter. The use of the Kalman filter methodology to model dynamics of brand equity has the following advantages. First, the approach allows us to understand the impact of marketing instruments both in the short-term and in the long-term. For example, we decompose the short-term effect of sales promotions into two components: (i) a positive effect that induces consumers to switch to the promoted brand, and (ii) a negative effect on the brand's equity that carries over from period to period. Although Jedidi et al. (1999) do differentiate between the short-term and long-term effects of sales promotions; the decomposition of short-term effects of sales promotions on consumer utility versus brand equity is not feasible using their methodology. Second, the recursive nature of the Kalman filter algorithm allows us to obtain estimates of brand equity for each period and not just infer changes in brand equity as influenced by advertising and sales promotions. Moreover, our approach integrates dynamics in the form of a state space formulation into the random coefficients logit demand model using aggregate data that are now popular in the New Empirical Industrial Organization (NEIO) literature (for example, Berry, Levinsohn, and Pakes 1995; Nevo 2001) and the related literature in marketing (for example, Chintagunta 2002; Sudhir 2001).[1]

[1] As is common in this literature, we also correct for the endogeneity of marketing mix variables such as price.

Given the estimates from the demand model, we compute the equilibrium advertising and sales promotion levels for each brand in the category while taking into account: a) the expected discounted value of the future profits, b) the effect of competitive strategies, and c) the competitive state of the market. To this end, following Dube, Hitsch, and Manchanda (2005), we compute the Markov Perfect Equilibrium (MPE) advertising and sales promotion strategies, which use dynamic programming based numerical solution techniques (Pakes and McGuire 1994). However, in addition to the optimal advertising budget allocation that Dube et al. (2005) consider, we also investigate the impact of sales promotions on the long-term profitability of a brand and its implications for optimal sales promotion budgets for myopic versus forward-looking decision makers.

We perform our analysis in two stages. In the first stage, we estimate the demand side parameters using store-level sales data. To this end, we develop and estimate a state-space model based on the Kalman filter that captures the dynamics of brand equity as influenced by marketing actions such as a brand's advertising expenditures and spending on sales promotions. In the second stage, we use these demand side estimates to compute the MPE advertising and sales promotion expenditures. First, we investigate the equilibrium advertising and sales promotion levels when the decision makers are myopic and consider only the current period profits. We then compare these levels with the case when the decision makers are forward-looking and maximize the expected discounted value of future profits, which is the ideal scenario. A comparison of the current strategy profiles with these two scenarios provides interesting insights into how the optimal advertising and promotion budgets depend on the correct formulation of the objective function.

We carry out our empirical analysis in the orange juice category using store- level sales data from a retail chain in the Chicago market. Our demand side estimation reveals that in the short-term, sales promotions have a positive effect on consumers' utility and they may induce consumers to switch to the promoted brand. However, there is also an adverse impact of sales promotions on brand equity that carries over from period to period. In the short-term, the magnitude of this negative effect is smaller than the positive effect of sales promotions caused by an increase in consumer utility. As a consequence, while the net short-term effect of sales promotions is positive, their long-term impact on a brand's market share and profitability can be negative due to their adverse impact on brand equity. As expected, advertising spending has a positive effect on brand equity. On the supply side, the results indicate the following: (i) under the myopic case, it will be optimal to increase the promotion budgets for both brands; (ii) however, the optimal forward-looking promotion levels are significantly lower than the actual levels for both brands; (iii) the myopic as well as the forward-looking advertising levels for both brands are lower than the actual levels. Hence, even if one were to consider the long-term effect of advertising, reducing the advertising budgets may be optimal. Overall, the forward-looking advertising and promotion levels imply that it is optimal for both brands to reduce their advertising and promotion levels.

The rest of the paper is organized as follows: We first review the related literature. Next, we present the state space model for estimating the demand side parameters and the details of the supply side model for estimating the MPE advertising and promotion strategy profiles. We then discuss the estimation of the demand side and the supply side models. Next, we describe the data and the operationalization of the variables. Subsequently, we present our empirical results based on the orange juice category and discuss the equilibrium advertising and sales promotion strategy profiles under various scenarios. Finally, we provide some concluding comments.

2. RELATED LITERATURE

As discussed in the previous section, we develop a framework for obtaining equilibrium advertising and sales promotion strategies when both these variables have a long-term effect on a brand's performance. In this regard, this paper is closely related to two recent papers in the marketing literature, Naik, Raman, and Winer (2005) and Dube, Hitsch, and Manchanda (2005). Similar to this study, both these papers investigate the optimal budgeting for marketing instruments when they have long-term effects on a brand's profitability. In this section, we discuss how this paper builds on these two papers.

The optimal allocation of the advertising and promotion budgets is an issue of practical importance. While promotions may have a strong positive effect on a brand's performance, they may also have some detrimental effects that need to be accounted for while allocating the marketing budget. In order to address this issue, Naik, Raman, and Winer (2005) extend the Lanchester model by incorporating interaction effects between advertising and sales promotions in addition to modeling their main effects. This allows for a marketing instrument to either amplify or attenuate the effectiveness of another marketing instrument. In their empirical application, they find that while the main effects of advertising and sales promotions are positive, there is a negative interaction between them implying that promotions decrease the effectiveness of advertising campaigns. The marketing mix algorithm they develop thus incorporates this tradeoff in addition to modeling strategic foresight amongst competing firms.

In deriving the equilibrium advertising strategies, Dube et al. (2005) use a logit demand model that accounts for the dynamic effects of advertising. One of the advantages of this demand system is the ability to explain the optimality of pulsing advertising strategies, which was one of the objectives of their study. However, the flexibility of the logit model implies that one cannot obtain analytical solutions, especially for closed-loop equilibria.[2] Correspondingly, they use the Markov Perfect Equilibrium (MPE) solution concept wherein the optimal advertising strategies are computed numerically.

In our application, we use a logit demand model to investigate the long-term effects of two marketing instruments, advertising and sales promotions, on a brand's profitability. There are a number of advantages to using the logit demand model as in Dube et al. (2005) as opposed to the Lanchester model. First, the logit model is derived from the behavioral assumption that consumers maximize utility while choosing an alternative. Hence, it is better suited for modeling the impact of marketing actions on consumers' utility, which in turn affects aggregate demand. For example, consider the long-term effect of promotions on demand. Extant research reveals that promotions may have a detrimental effect on brand equity (see, for example, Jedidi et al. 1999), even if they have a positive short-term effect on sales. We

[2]Chintagunta, Kadiyali, and Vilcassim (2005) obtain an analytic expression for open-loop advertising strategies using a logit demand model.

cannot discern such differential short-term and long-term effects using a Lanchester model. Using the logit model, we can decompose the effect of promotions into two components – a short-term positive effect and a long-term negative effect through its effect on the dynamic brand equity. Such decomposition can help in explaining why myopic decision makers are likely to choose higher than optimal promotion levels and lower than optimal advertising levels – a practice for which managers are routinely criticized. Second, the S-shaped sales response implied by the logit model has several desirable properties such as being able to explain pulsing advertising strategies that are typically observed in consumer packaged goods markets (Dube et al. 2005). Third, a logit demand model is flexible enough to account for consumer heterogeneity. Finally, the logit formulation is routinely used in the marketing literature to model demand.

Substantively, as in Naik, Raman, and Winer (2005), our problem deals with the issue of optimally allocating the marketing budget between advertising and sales promotions when both these instruments have long-term effects. However, we use an approach similar to Dube et al. (2005). Nevertheless, in addition to the equilibrium advertising strategies that they investigate, we also compute the equilibrium promotion levels when such promotions may have a long-term impact. Hence, we build on Dube et al. (2005) by considering the tradeoff between advertising and sales promotions when temporary price cuts and promotional deals may have long-term consequences.

3. MODEL

3.1. The Demand Model

We consider a market with utility-maximizing households who while shopping at store s (s=1, 2, …,S) in a given period t may choose to purchase a brand j (j=1,2,…,J) within a category or may purchase an outside good (equivalent to not purchasing in the category, denoted by j = 0). The presence of the outside alternative in our model allows for the potential market expansion and contraction. We represent the utility that household h derives from buying brand j, j = 1, 2,…,J, in store s, s=1, 2, …, S, in period t as,

$$U_{hjst} = \beta_{0hjst} + \alpha_h P_{jst} + \beta X_{jst} + \gamma_h Pr_{jst} + \xi_{jst} + \varepsilon_{hjst}, \qquad j = 0, 1, 2, \ldots, J, \; s=1, 2, \ldots, S, \qquad (1)$$

where β_{0hjst} is the utility that household h derives from brand-name j in store s at time t, X_{jst} is a vector of factors that influence the household's utility including demand drivers such as seasonal factors,[3] P_{jst} and Pr_{jst} are the regular price and promotion, respectively of brand j in store s in period t, ξ_{jst} is the mean utility to consumers from brand j in store s in period t due to unobserved variables, and ε_{hjst} represents the idiosyncratic preference of household h for brand j in store s in period t. In Equation (1), we assume that consumers in each period will choose to purchase one of the J brands or settle for the outside good depending on the utility that they expect to derive from each choice alternative. Their purchase choice is thus based on a consideration of the characteristics of alternative brands, the regular prices of alternative brands, promotional deals, seasonality and of course, the brand names.

In Equation (1), the term ξ_{jst} captures the effects of variables unobservable to the econometrician but observed by the firm and the consumers. Such variables may include unobserved product characteristics such as a brand's shelf location, the amount of shelf space devoted to a particular brand, and other brand demand drivers at the store that vary over time (Chintagunta 2002). Since firms observe ξ_{jst}, they may incorporate this information in setting the price for period t thus posing endogeneity issues that will be addressed later in this paper. The coefficient vector β contains consumer taste parameters for demand shifters such as seasonality, while α_h and γ_h represent the responsiveness of household h to regular price and promotions, respectively. The parameter α_h and γ_h are random coefficients such that

$$\begin{aligned} \alpha_h &= \alpha + \Delta\alpha_h \\ \gamma_h &= \gamma + \Delta\gamma_h \end{aligned}, \qquad (2a)$$

[3] Although seasonal factors are not expected to vary by store, we have retained the store subscript for the sake of generality.

where α and γ are the mean response parameters, and $\Delta\alpha_h$ and $\Delta\gamma_h$ are the corresponding household specific deviations from these mean response parameters. Similarly, the utility that consumer h derives from the brand name in store s at time t, $\boldsymbol{\beta_{0hst}} = (\beta_{0h1st}, \ldots, \beta_{0hJst})$ is given by

$$\boldsymbol{\beta_{0hst}} = \boldsymbol{\beta_{0st}} + \Delta\boldsymbol{\beta_{0h}}, \tag{2b}$$

where $\boldsymbol{\beta_{0st}}$ is the vector of mean brand equities in store s at time t across consumers and $\Delta\boldsymbol{\beta_{0h}}$ is the corresponding deviation of household h's brand equities from the corresponding mean levels. We assume that the heterogeneity parameters, $(\Delta\boldsymbol{\beta_{0h}}, \Delta\alpha_h, \Delta\gamma_h)$ are drawn from a multivariate normal distribution with covariance Σ. Hence, we have $(\Delta\boldsymbol{\beta_{0h}}, \Delta\alpha_h, \Delta\gamma_h) \sim \mathrm{N}(\mathbf{0}, \Sigma)$. In theory, it is possible to allow for non-zero covariances between all the heterogeneity parameters. However, such an unrestricted model will require the estimation of a very large number of parameters. Hence, for the sake of parsimony, we allow for correlations between the equities of alternative brands but restrict the remaining parameters to be uncorrelated. For the purpose of identification, we set the mean utility of the outside good to be equal to 0. Hence, $U_{h0st}, = \varepsilon_{h0st}$.

Following Berry et al. (1995), we can rewrite U_{hjst} in Equation 1 as follows:

$$U_{hjst} = \delta_{jst} + \mu_{hjt} + \varepsilon_{hjst}, \tag{3}$$

where the mean utility level, δ_{jst}, for brand j and the household-specific random coefficient component μ_{hjst} are defined as follows:

$$\delta_{jst} = \beta_{0jst} + \beta X_{jst} + \alpha P_{jst} + \gamma_h Pr_{jst} + \xi_{jst}, \tag{4}$$

$$\mu_{hjst} = \Delta\beta_{0h} + \Delta\alpha_h P_{jst} + \Delta\gamma_h \Pr_{jst} . \tag{5}$$

Our definition of the utility of the outside good, U_{h0st}, implies that both δ_{0st} and μ_{h0st}, are normalized to be zero. Further, we assume that $\xi_{jst} \sim N(0, \sigma^2_{\xi s_j})$.

The mean brand equity, $\boldsymbol{\beta_{0jst}}$, in Equation 4 captures the incremental utility that the average consumer derives from brand name j in store s at time t with respect to the outside alternative and this has been previously used in the marketing literature as a measure of brand equity (for example, Kamakura and Russell 1993; Jedidi, Mela, and Gupta 1999). This measure is consistent with the operationalization of brand equity in Srinivasan (1979) and Kamakura and Russell (1993) as the component of a consumer's utility not explained by the objectively measured attributes and the marketing mix variables.[4] Conceptually, this brand equity term, is similar to the goodwill formulation used by Dube et al. (2005). However, unlike in their case where the logarithmic transformation of goodwill affects indirect utility, the brand equity term enters Equation 4 linearly in our formulation.[5] We choose to use a linear specification for the following reasons: (a) a linear specification is consistent with the prior work in the literature on how brand equity enters the utility equation (see, for example, Kamakura and Russell 1993; Jedidi et al. 1999). (b) a logarithmic transformation of the brand equity will make Equation 4 non-linear, which is inconsistent with the assumptions in our standard linear Kalman filter estimation.

Note that we allow for the brand equities to vary over time and thus, capture the dynamics of brand equity over time (see Equation 1). We model the dynamics in a brand's equity by allowing it to be a sum of three components as follows:

$$\beta_{0jst} = \beta_{0s} + \overline{\beta}_j + \zeta_{jst} \quad , \qquad j = 1, 2, \ldots, J, \quad \text{(6a)}$$

where, β_{0s} is a store-specific intercept that allows each store to have different mean equity for each brand, $\overline{\beta}_j$ is the time (and store) invariant component of the mean equity of brand j, and ζ_{jst} is the dynamic component of the brand equity of brand j. Note that the store-specific intercepts, β_{0s}, do not vary by brand and capture any store-specific differences in category consumption that may affect all the brands. Consistent with the notion that advertising and sales promotion have an impact on brand equity over time (see, for example, Jedidi, Mela, and Gupta 1999), we model the dynamic component of the mean brand equities as

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[4] The intercept-based measure is just one of the ways of measuring brand equity. Alternatively, one can measure brand equity using consumer brand knowledge (Keller 1993), product-market performance (Ailawadi, Lehmann, and Neslin 2003), or the shareholder value (Simon and Sullivan 1993).
[5] Dube et al. (2005) use a logarithmic transformation in order to ensure that the optimal advertising levels are bounded. In our application, the linear specification does not lead to infinite advertising levels.

$$\zeta_{jst} = R\zeta_{0jst-1} + \Gamma_j^{Ad} \ln(1 + Ad_{jt}) + \Gamma_j^{\Pr} \Pr_{jst} + e_{jst} \ , \qquad (6b)$$

where Ad_{jt} is the level of advertising for brand *j* in time period *t*, and Pr_{jst} is the level of promotion for that brand *j* in store *s* at time *t*.[6] The parameters Γ_j^{Ad} and $\Gamma_j^{\Pr}$ capture the contemporaneous effects of advertising and sales promotions on brand *j*'s equity respectively. The parameter *R* captures the extent to which brand equity carries over from period to period and can be interpreted as a measure of persistence in brand equity. The error term, e_{jst}, captures the variation in customer preference for brand *j* at time *t* that is not explained by either the carryover of brand equity from the previous period or the advertising and sales promotion variables. We assume that $e_{jst} \sim N\ (0, \sigma^2_{es_j})$. The logarithmic transformation of advertising captures the diminishing effect of advertising on brand equity.

A comment or two about our representation of the dynamic component of brand equity in Equation 6b is in order. First, since brand equity carries over from period to period, the impact of advertising and sales promotion on brand equity will also carry over from period to period. Such a formulation is consistent with the finding that advertising and sales promotion have long-term effects on brand equity (for example, Jedidi, Mela, and Gupta 1999) and the extent of this carryover will depend on the magnitude of the parameter *R* with higher values of *R* implying a higher level of carryover. Second, since the long-term effects of advertising and sales promotion are realized because of the carry over in brand equity, an implication of our model is that brand equity acts as a mediating variable in determining the long-term consequences of marketing actions. As mentioned, this is in contrast to most of the extant literature, with the exception of Jedidi, Mela, and Gupta (1999), which models the long-term implications of marketing actions with sales or market share as the mediating variable (see, for example, Dekimpe and Hanssens 1999). We suggest that, using brand equity as a mediating variable can help us better understand if there are any adverse long-term consequences of some marketing actions such as sales promotions even if they may have a beneficial effect in the short-term. Third, the fact that brand equity

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[6] Although we include advertising at the geographical region where the store is located, due to constraints in obtaining such data, we use a common advertising variable for all the stores. However, if one had access such micro level data, they can be used in the estimation.

carries over from period to period implies a positive state dependence (inertia) in consumer purchases from period to period, which is generally considered to be a measure of brand loyalty. Hence, according to our specification, brand loyalty is an outcome of brand equity.

Note that in Equation 6b, we do not observe the values of brand equity in each time period *t*, but need to estimate them. In order to obtain estimates of this unobserved brand equity for each period, we use the Kalman filter algorithm, which has been used extensively in control engineering and has been recently applied in the marketing literature (for example, Xie, Sirbu, and Wang 1997; Putsis 1998; Naik, Mantrala, and Sawyer 1998; Akcura, Gonul, and Petrova 2004). The Kalman filter is a recursive algorithm that is used to obtain efficient estimates of an unobserved state variable (which happens to be brand equity in our case) in each period based on the information observed in that period. The recursive nature of the algorithm enables updating of the estimates of the unobserved state variables and is thus useful in obtaining time varying estimates of unobserved quantities such as brand equity.[7]

Assuming that the error terms ε_{hjst} are distributed i.i.d. with a type I extreme value density function, the probability of household *h* buying brand *j* in store *s* at time *t* is given by

$$prob_{hjst} = \frac{\exp(\delta_{jst} + \mu_{hjst})}{1 + \sum_{j'=1}^{J} \exp(\delta_{j'st} + \mu_{hj'st})} \quad . \tag{7}$$

However, since we use aggregate data, we do not observe purchase probabilities at the individual level. Hence, we need to aggregate the probabilities from Equation 7 across all households to obtain the market share for brand *j*. We do this by integrating Equation 7 across all households to yield the market share s_{jst} in store *s* for period *t* as

$$s_{jst} = \int prob_{hjst}\, f(\nu_h)\, d\,\nu_h. \tag{8}$$

[7] We present a detailed description of the estimation procedure in the appendix.

In the above equation, *f(.)* is the joint distribution of the elements of ν_h, the heterogeneity parameters. Given random draws from the distribution of ν_h, the integral in Equation (8) can be computed numerically.

3.2. Modeling Firm Decisions: The Supply Side

The tradeoff between advertising and sales promotions traditionally derives from the fact that while advertising has a very small short-term effect, it has a lasting effect on a brand's performance because of the carryover in brand equity. On the other hand, although sales promotions may have a positive short-term effect, they may have a negative effect on brand equity that carries over from period to period. In this section, we characterize the advertising and sales promotion decisions that firms need to make given these trade offs. While the demand side model was calibrated at the store level, we model the supply side (firm decisions) at the aggregate market level. Correspondingly, we do not include the effect of the store level intercepts, β_{0s}, in the supply side model and estimation. Further, we drop the store subscript *s* in this section.

We assume that the exact timing of the advertising and sales promotion decisions is as follows. At the beginning of each time period, each firm observes the brand equity levels of all the firms, β_{0jt}, j = 1, 2, …, *J*.[8] In addition, the firms know the quarter of the year they are in and the associated demand changes that they can expect during the quarter due to seasonality. The brand equities of the various firms in the market along with seasonality (as characterized by the quarterly dummies) constitute the state of the market. Formally, we define the state vector at time $t, S_t = (\beta_{01t}, \beta_{02t}, ..., \beta_{0Jt}, Q_t)$, where Q_t indicates the quarter of the year to which *t* belongs. The firms make their marketing decisions (advertising and sales promotions) based on the observed values of these state variables.

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[8] The only time varying component of brand equity is the dynamic component, ζ_{jt}. Hence, computing the strategy profiles based on changes in the brand equity is tantamount to computing them based on changes in the dynamic component. Hence, the actual computation of the strategy profiles is based on the dynamic component, ζ_{jt}.

As discussed in Section 3.1 (Equation 6b), our model implies that the effect of sales promotions stems from two sources: (a) a direct effect on the utility that only has a contemporaneous effect, and (b) an indirect effect through its impact on the brand's equity that carries over from period to period. On the other hand, the effect of advertising stems only from its effect on brand equity. It is the carryover of brand equity that induces the dynamics in our model, which, in turn, implies that the advertising and sales promotion decisions have long-term consequences on consumer demand. While Equation 6b characterizes the dynamics of brand equity from period to period, similar to Dube, Hitch, and Manchanda (2005), we assume the following time ordering of events. The strategy profile chosen at the beginning of the period affects equity of the brand and creates the augmented brand equity, β_{0jt}^{a}. Specifically,

$$\beta_{0jt}^{a} = \overline{\beta}_j + \zeta_{jt}^{a} ,$$

$$\zeta_{jt}^{a} = \zeta_{jt} + \Gamma_j^{Ad} \ln(1 + Ad_{jt}) + \Gamma_j^{\Pr} \Pr_{jt} , \qquad (9)$$

where ς_{jt} is the dynamic component of brand equity at the beginning of the period t. It is this augmented brand equity that affects product demand. Over time, the augmented brand equity, β_{0jt}^{a}, and its dynamic component, ζ_{jt}^{a}, depreciate stochastically as follows:

$$\begin{aligned} \zeta_{jt+1} &= R\zeta_{jt}^{a} + e_{jt+1} \\ \beta_{ojt+1} &= \overline{\beta}_j + \zeta_{jt+1} \end{aligned} . \qquad (10)$$

Correspondingly, β_{0jt+1} is the brand equity at the beginning of period $t+1$. Hence, β_{0jt+1} will be a part of the state vector S_{t+1}, based on which the advertising and sales promotion decisions for period t+1 are made. Taken together, Equations 9 and 10 are consistent with the characterization of the dynamics in brand equity in Equation 6b.

Equation 8 provides the link between market share and the state as well as the control variables. Given this market share and a definition of the market size, we can compute the per period profit for brand j as

$$\tilde{\pi}_{jt} = (W_{jt} - c_j - \frac{\Pr_{jt}}{\rho})M_t s_{jt}(S_t, Ad_{jt}, \Pr_{jt}, \xi_{jt}) - Ad_{jt}, \quad (11)$$

where W_{jt} is the wholesale price of brand j at time t, c_j is the marginal cost of brand j, M_t is the market size at time t, and ρ is the retail pass through for the promotions. In our application, we restrict the decision variables to advertising and sales promotion spending levels and do not include price. Hence, the values of the retail and wholesale prices, retail pass through, market size, and the marginal cost are fixed while determining the optimal advertising and promotion levels. We discuss the values at which these variables are set in Section 5.2. As in Dube et al. (2005), we assume that the advertising and promotion decisions are made prior to the realization of the demand shocks, ξ_{jt}.[9] Hence, these decisions are made based on the expected profits, which is defined as

$$\pi_{jt} = \int (W_{jt} - c_j - \frac{\Pr_{jt}}{\rho})M_t s_{jt}(S_t, Ad_{jt}, \Pr_{jt}, \xi)p(\xi)d\xi - Ad_{jt}. \quad (12)$$

The strategy profile for all the firms is $\sigma(S_t) = (\sigma_1(S_t),...,\sigma_J(S_t))$, where $\sigma_j(S_t) = (Ad_{jt}, \Pr_{jt})$, lists the decision rules of all the firms. Given the state vector S_t and the corresponding strategy profile, the expected present discounted value of profits for firm j is

$$V_j(S_t \mid \sigma) = E[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \pi_j(S_\tau, \sigma_j(S_\tau)) \mid S_t], \quad (13)$$

where δ is the discount factor. Firms make the advertising and sales promotion decisions that would maximize their expected present discounted profits. The computation of the expectation in Equation 13 requires knowledge regarding the evolution of the state variables. From Equations 9 and 10, it is clear that the equity of brand j at time t, β_{0jt}, follows a Markov process with transition density $p(.\mid \beta_{0jt-1}, Ad_{jt}, Pr_{jt})$. Given the definition of the time period as a quarter, the Markov transition density for the other state variable, Q_t, is obvious. However, unlike the case of brand equity, the transition probability for this state

[9] Alternatively, one can argue that the advertising and promotion decisions are made based on the expected demand shocks. Under such a scenario, as in Nair (2005), we need to include the demand shocks, ξ_{jt}, as state variables, which would significantly complicate the estimation.

variable will be non-stochastic and independent of the firm's advertising and sales promotion decisions. Given our assumption that the error terms in the system equation (Equation 6b), e_{jt}, are i.i.d., we can write the transition density of the state vector as

$$p(S_{t+1} \mid S_t, Ad_t, \Pr_t) = \prod_{j=1}^{J} p(S_{jt+1} \mid S_{jt}, Ad_{jt}, \Pr_{jt}). \qquad (14)$$

We assume that the firms make their advertising and sales promotion decisions only based on the information contained in the current state vector. Hence, the strategies are not time dependent. The value function in Equation 13 satisfies the Bellman equation,

$$V_j(S \mid \sigma) = \max_{Ad_{jt}, \Pr_{jt} > 0} \{\pi_j(S, \sigma_j(S), \sigma_{-j}(S)) + \delta \int V_j(S' \mid \sigma) p(S' \mid S, \sigma_j(S), \sigma_{-j}(S)) dS'\}. \quad (15)$$

In the above equation, $\sigma_j(S) = (Ad_{jt}, Pr_{jt})$ corresponds to the strategy profile of brand j given the state S and $\sigma_{-j}(S)$ is the strategy profile of all the other brands. Hence, given its strategy profile, $\sigma_j(S)$, brand j makes an assumption about the competitive strategy profile, $\sigma_{-j}(S)$. These help in determining the transition probabilities, $p(S_{t+1} \mid S_t, Ad_t, \Pr_t)$. Hence, the strategy profile for brand j given the state vector S, $\sigma_j(S)$, is obtained by maximizing the value function defined by Equation 15. However, the right hand side of Equation 15 is defined conditional on a specific guess about the competitive strategy profile, $\sigma_{-j}(S)$. Hence, the strategy profile $\sigma_j(S)$ that maximizes the right hand side of Equation 15 is the best response by brand j given the assumption about the competitive strategy profile, $\sigma_{-j}(S)$. In equilibrium, these assumptions about the competitive strategy profile will coincide with the optimal strategy profile of each of the competitors. We discuss the computation of the equilibrium strategies in the next section.

4. ESTIMATION

4.1 Demand Estimation

The objective of our estimation is to recover three sets of parameters: (i) parameters $\Theta_1 = \{ \beta_{0s}$, $\overline{\beta}_j$, $\alpha, \beta, \gamma\}$ that affect the mean utility of brand j at time t in the observation Equation 4, (ii) parameters $\Theta_2 = \{R, \Gamma_j^A, \Gamma_j^P\}$ that capture the dynamics of brand equity in the system equation (Equation 6b), and (iii) parameters Θ_3 that capture consumer heterogeneity. We proceed with the estimation as follows:

Step 1: For a given set of heterogeneity parameters, Θ_3, we evaluate the integral in Equation 8 numerically. Using the numerically computed values of the market shares, we obtain the mean utility of brand j at time t using the contraction-mapping algorithm proposed by Berry et al. (1995).[10]

Step 2: Next, we express the mean utilities thus recovered as a linear function of the observed variables and the set of parameters Θ_1 in Equation 4. This corresponds to the *observation equation* in the state space framework. The equation that captures the dynamics of brand equity (i.e., Equation 6b) corresponds to the *system equation* in the state space framework. This two-equation system comprising of the observation equation (Equation 4) and the system equation (Equation 6b) is estimated using a Kalman filter algorithm that is iterated until convergence is reached to obtain efficient estimates of the equity of brand j at time t, β_{0jst}, as well as the error term in the observation equation, ξ_{jst}. This error term is fed into a generalized method of moments (GMM) objective function, which is iterated until convergence is attained, as discussed below.

We use GMM to estimate the parameters $\{\Theta_1, \Theta_2, \Theta_3\}$. Sriram, Chintagunta, and Neelamegham (2005) use a similar estimation procedure based on GMM and demonstrate its ability to recover the true parameters in Monte Carlo simulation Studies. The GMM estimation makes use of the fact that the instruments are not correlated with the errors and hence, the instruments, and the errors should be orthogonal (or at least as orthogonal as possible). The GMM objective function to be minimized is given by,

[10] Berry et al. (1995) prove that for each value of the observed market shares, given the identifying restriction that the mean utility of the outside alternative is equal to 0, the contraction mapping algorithm will yield unique values of the mean utilities.

$$\xi' Z W^{-1} Z' \xi \,, \qquad (16)$$

where the wings of the quadratic form are the moments $\xi' Z$, which are basically the residuals interacted with the instruments and W is a consistent estimate of the variance-covariance matrix of the moments. In the iterative algorithm used to minimize the objective function in Equation 16, the ξ_{jst} terms are computed for each iterated estimate of the heterogeneity parameters (the elements of the Cholesky decomposition of the covariance matrix Σ), by "inverting" the observed market shares in Equation 8 to yield estimated values of the mean utilities, δ_{jst}.[11] Given the estimated values of δ_{jst}, the parameters, β_{0jst}, $\boldsymbol{\beta}$, and α_q are estimated in a straightforward fashion, leading to an estimate of ξ_{jst} as the residual in Equation 4. The estimate of ξ_{jst} is then used to compute the GMM objective function for that iteration. Interested readers are referred to Nevo (2001) for further details regarding the estimation algorithm.

4.2. Supply Side: Computing the Equilibrium Strategies

As stated earlier, in equilibrium, for each value of the state vector, the strategy profile for brand j obtained based on its assumption regarding the strategy profiles of its competitors by maximizing the right hand side of Equation 15 should coincide with the strategy profiles of each of the competitors obtained similarly. We obtain the Markov Perfect Equilibrium (MPE) strategies numerically given the demand side estimates. In computing the Markov Perfect Equilibrium strategies, we use brand equity and seasonality (quarter) as the state variables. We compute the equilibrium using the following steps:

Step 1: Start with some initial guesses of the strategy profiles of all the brands.

Step 2: Given the strategy profiles, compute the value function for each value of the state vector that satisfies the right hand side of Equation 15. Since brand equity is a continuous variable, as in Nair (2005), we compute the value function at a few grid points and approximate the value function as a tensor product of a Chebyshev polynomial basis in each continuous state dimension (Judd 1999).

[11] This inversion is achieved through a contraction mapping procedure as in Berry et al. (1995).

Step 3: Given these value functions, and the competitive strategy profiles, determine the optimal strategy profile for each firm that would maximize the right hand side of Equation 15.

We iterate Steps 2 and 3 until the differences in the strategy profiles from two successive iterations are lower than a preset level of tolerance for all the firms. At this point, we can infer that the competitive strategy profile based on which a firm's optimal strategy profile was obtained coincides with the best strategy profile for each of its competitors. Note that this is an equilibrium in pure strategies only. However, there is no guarantee for the existence or uniqueness of this equilibrium. The convergence of the above algorithm is sufficient to prove the existence of an equilibrium for a specific parameterization of the model (Benkard 2000). However, a more difficult problem is the existence of multiple equilibria. We checked for the presence of multiple equilibria by starting with different values of initial strategy profiles. Each time, we converged at the same set of equilibrium strategy profiles. Hence, the presence of multiple equilibria does not appear to be a problem in our case.

5. DATA DESCRIPTION AND OPERATIONALIZATION OF VARIABLES

5.1 Data Description

Given that our main objective is to capture the dynamics of brand equities, we need a database that spans an extended time period. This is especially true since brand equity is an enduring construct and is unlikely to fluctuate on a weekly basis. An extended data observation time period is also required to obtain stable parameter estimates using the Kalman filter methodology. The *Dominicks Finer Foods* database made available by the University of Chicago meets these requirements. The data span almost eight years from 1989 to 1997 and consist of weekly observations of sales, shelf prices, and the possible presence of sales promotions (coupons, bulk buy, or a special sale) by individual item (UPC) and daily store traffic for most of the 96 stores operated by Dominicks Finer Foods in the Chicago area.[12] We selected a random sample of 32 stores for our estimation. We aggregated the sales data at the quarterly level for each store. We selected a quarter as the time period for the level of aggregation for a number of

[12] UPC stands for Universal Product Code.

reasons. Given brand equities are expected to be relatively stable over time (Ailawadi, Lehmann, and Neslin 2003), we do not expect significant perceptible changes on a weekly basis. On the other hand, we need sufficient number of observations over time in order to be able to estimate the dynamics in brand equities. Moreover, our advertising data are at the quarterly level. This aggregation at the quarterly level yielded 30 periods of data, which is sufficient to estimate the dynamics of brand equities. We supplement the store data with data on the quarterly national advertising expenditures of brands, obtained from *Leading National Advertisers'* Ad Dollar summary.

We perform our empirical analysis using store-level scanner data in the orange juice category. Tropicana and Minute Maid, the two major brands in this category, together account for about 63% of the total market share. Our analysis is based on the two popular sizes, viz., 64 oz. and 96 oz., of these two brands. Together, these two sizes account for about 95% of the total orange juice sales of these brands. Descriptive statistics for the brands included in our analysis are displayed in Table 1. The leading brand in terms of market share is Tropicana with an average market share of over 41%. It also commands a price premium over Minute Maid in terms of the price charged per oz. The level of sales promotions as well as the average advertising expenditure is higher for Tropicana compared to Minute Maid.

5.2. Operationalization of Variables

5.2.1. Variables Used in the Demand Side

(a) Marketing Mix Variables

The marketing mix variables include a brand's regular price and the level of sales promotion as well as advertising during a given quarter. We operationalized the regular price variable as the weighted (by unit sales) average non-promoted price per oz. of the UPCs of the brand. We define the non-promoted price of each UPC as the average price of the UPC when it was not on sale during that quarter. Correspondingly, the retail price was computed as the weighted average price per oz. of the UPCs offered by the brand in that quarter. The promotion variable was operationalized as the difference between the regular price and the retail price. Note that this definition of promotions captures both frequency and depth of promotions. In addition to these two variables, we include three seasonal dummies for the first three quarters (with the last quarter of the year set to zero) as demand shifters in order to capture seasonality. We convert the national advertising to regional advertising by assuming that the firms allocate their advertising expenditures across different regions based on their population. Since the Chicago metropolitan area accounts for approximately 3.26% of the national population, we assume that the same percentage of the national advertising was spent in the Chicago region. Note that this advertising includes the advertising for all the products manufactured by the brands including frozen and refrigerated non-orange juice drinks. We deflate the advertising data by the consumer price index in order to account for inflation.

(b) Outside Alternative

Our estimation requires the definition of an outside good, or no-purchase alternative. We assume that each household has the potential to consume 64 oz. of orange juice every week and then multiply the store traffic in a quarter by the quarterly consumption rate to define the market size. We then subtract the sales of the brands under consideration to compute the "sales" of the outside alternative. The respective shares are then computed from the sales of the brands and the market size as defined above. This approach is similar to that used by Chintagunta (2002).

(c) Instrumental Variables

We allow for endogeneity of price through the use of instrumental variables. The instrumental variables chosen should be such that they are strongly correlated with price, but uncorrelated with the error term (ξ_{jst}.). A reasonable choice is the wholesale prices since they are likely to be correlated with the retail prices but are unlikely to be chosen on a week-to-week basis by the manufacturers, who typically make quarterly sales plans in these categories. Wholesale prices have been used as instruments in several previous studies in marketing (e.g., Chintagunta 2002; Chintagunta, Dube, and Singh 2003). Since a retailer's margins may depend on the brand and other product characteristics, we interact the wholesale price with the brand dummies. We treat sales promotions as exogenous, since such decisions are generally made on a quarterly basis and often require a lead-time of several weeks for effective implementation. Indeed, based on conversations with local chain managers, Chintagunta, Dube, and

Singh (2003) report that while the sales promotion calendar is determined in advance, pricing decisions are made subsequently, conditional on the promotional calendar. Hence, they argue that sales promotions can be treated as exogenous, unlike prices. Several other studies also treat sales promotions as exogenous (see, for example, Chintagunta 2002).

5.2.2. Variables in the Supply Side

As mentioned earlier, we restrict our investigation to the determination of the optimal advertising and promotion levels. Correspondingly, we compute the equilibrium policies in the supply side by fixing the regular prices (both retail and wholesale), market size, marginal cost, and retail pass through. We performed the supply side analysis with the Chicago metropolitan area as the basis. Following the assumption we made in the demand side estimation, we assumed that each household has the potential of consuming 64 oz. of orange juice every week. We then multiplied this by the number of households in the Chicago area (roughly 3.16 Million) to obtain the total market size.[13] Inherent in this is the assumption that the responsiveness of the brands to the changes in their marketing mix as well as to the seasonal factors is the same across all the chains in the Chicago area. We fixed the retail and wholesale prices at the average values reported in Table 1. As in Jedidi, Mela, and Gupta (1999), we assume that the marginal cost is 30% of the wholesale price. As regards retail pass through, several studies (see, for example, Besanko, Dube and Gupta 2005) have documented that the retailers may not pass on all the price reduction given by a manufacturer. Correspondingly, based on the empirical findings in that paper, we assume a retail pass through of 82%.

6. RESULTS

6.1. The Demand Side

We estimated the model for the two leading brands in the orange juice category – Tropicana and Minute Maid. During the time period for which we have data, these two brands accounted for over 63%

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[13] We obtained this information from *Market Scope* for the year 1999.

volume sales in this category. Recall that one of the advantages of using the Kalman filter methodology is the ability to obtain estimates of brand equity for each time period.

Table 2 displays the parameter estimates from the demand side estimation. For ease of exposition, we divide the parameters into three groups: (i) the parameters in the observation equation (Equation 4), (ii) the parameters in the system equation (Equation 6b), and (iii) the heterogeneity parameters in Equation 5.[14] We observe in Table 2 that all the parameters in the observation equation are statistically significant at least at the 10 percent level in a two-tailed test. As expected, the price coefficient is negative ($p < .01$) and the sales promotion coefficient is positive ($p < .01$). We discuss the price and promotion elasticities implied by these estimates subsequently. The average brand equity over the 30 quarters and across all the stores was –3.40 for Minute Maid and –2.91 for Tropicana. Thus, Tropicana has higher brand equity than Minute Maid in the orange juice category. This is not surprising since Tropicana has a higher market share and also commands a significant price premium in the orange juice market. The average brand equity is negative because the mean utility of the outside alternative is fixed at 0 and the share of the outside alternative is significantly higher than that of the inside goods. The intrinsic component of brand equity that is invariant to marketing actions is –3.82 for Minute Maid and –3.45 for Tropicana.[15]

In the system equation, the time-varying component of the brand equity (the term ζ_{jst} in Equation 6a) for Minute Maid ranges from -0.83 to 1.39 across the 32 stores. Across all the 32 stores and over the 30 quarters, the Minute Maid's brand equity ranges from -4.65 to -2.43. Similarly, the time varying component of Tropicana's brand equity ranges from -1.18 to 1.95. Tropicana's brand equity ranges from -4.63 to -1.50 across all the 32 stores over the 30 quarters. The estimate of the parameter that

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[14] In addition, we estimate the standard deviations of the observation and the system equation errors. The observation equation error standard deviation was 1.653 and the system equation error standard deviation was 2.34×10^{-3}.

[15] In consideration of space constraints, we do not report the estimates corresponding to the store-specific intercepts in Table 2. The store-specific intercepts (the term β_{0s} in Equation 6a) range from a low of -0.136 to 0.477. The higher store-specific intercepts correspond to stores that exhibit higher consumption of Minute Maid and Tropicana refrigerated orange juice brands. Correspondingly, in stores that have lower store-specific intercepts, the consumption of the outside alternative (including the store brand) is higher.

captures the carryover of brand equity from period to period (LAG BE) is 0.773. This is consistent with our expectation that there should be a positive and significant carryover of brand equity.

We also observe in Table 2 that sales promotions of both Minute Maid and Tropicana have a negative effect on own brand equity; the adverse impact is, however, larger and statistically significant ($p < 0.1$) in the case of Tropicana. This negative effect of sales promotions on the two brands' equity is consistent with the attribution theory wherein consumers attribute their purchases to the offer of a promotional deal rather than their underlying preference for the brand (see, for example, Dodson, Tybout, and Sternthal 1978). Moreover, prior research has documented that the frequent use of sales promotions can lower the reference price and thus adversely affect the price premium that the brand can charge (see, for example, Kalwani, Rinne, Sugita, and Yim 1990; Kalwani and Yim 1992; Lattin and Bucklin 1989; Blattberg et al. 1995). Hence, we expect that the premium priced Tropicana will be more prone to such an adverse effect. Consistent with the prior research, our demand side results reveal that the magnitude of the detrimental effect of sales promotions is higher for Tropicana than for Minute Maid. These results highlight a key advantage of estimating the dynamics of brand equity using the Kalman filter methodology, namely, a capability to isolate the effect of promotions on the utility versus that on brand equity. Note that despite their adverse impact on brand equity, the net short-term effect of sales promotions on mean utility and hence, market share, is positive for both brands.[16] Finally, as expected, we observe in Table 2 that advertising has a significant positive effect on the equity of both brands. The effect is higher for Tropicana than for Minute Maid. This is consistent with Keller's (1998) argument that the advertising of brands with greater equity is more effective. We consider the long-term effects of sales promotions and advertising later in this section.

Turning now to the heterogeneity parameters, the estimates imply significant consumer heterogeneity only in the price coefficient. Although not significant, the covariance between the

[16] The total contemporaneous effect of sales promotions is obtained as the sum of the direct effect in the observation equation and the brand specific effect in the system equation

heterogeneity of Tropicana and Minute Maid is negative. This implies that consumers who have a higher preference for Tropicana tend to have a lower preference for Minute Maid and vice versa.

Based on these demand estimates, we computed the *market share elasticities* with respect to price, promotion, and advertising. Since promotions and advertising affect the dynamic component of a brand's equity (Equation 6b), in addition to having a contemporaneous effect, an increase in these variables will also have a long-term effect on a brand's equity. We present the implied market share elasticity estimates in Table 3.

The estimate of market share elasticity with respect to *price* for Tropicana is –1.63 and that for Minute Maid is –1.57. These values lie within the range of price elasticity estimates reported by Tellis (1988). The slightly higher price elasticity for Tropicana may be due to fact that Tropicana has a higher retail price than Minute Maid.[17]

The short-term market share elasticity with respect to *sales promotions* is positive for both brands and is larger for Tropicana (0.41) than for Minute Maid (0.28). This can be attributed to the higher promotion levels for Tropicana (see Table 1). This is despite the adverse effect of sales promotions on brand equity being higher for Tropicana. We also observe in Table 3 that the short-term sales promotion elasticity is smaller in magnitude than the price elasticity. While this may seem contrary to the findings in the literature that promotional elasticities exceed price elasticities (see, for example, Blattberg et al. 1995), a closer look at the magnitude of the price cut associated with sales promotions will shed some light on this apparent anomaly. From Table 1, we can see that the average promotion is about 10% of the regular price. Correspondingly, a 1% increase in promotion will be equivalent to a 0.1% decrease in price. Hence, the price cut associated with a 1% change in the level of the promotional variable is much lower in magnitude than a 1% reduction in regular price. Therefore, the promotional elasticities appear to be smaller in magnitude than the corresponding price elasticities.

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[17] Note that in a logit model, in the absence of any heterogeneity, the price elasticity of a brand is directly proportional to the price of the brand i.e., price elasticity of brand j at time t = $\alpha p_{jt}(1-s_{jt})$, where α is the estimated price coefficient. Given that the last term within parentheses is generally close to 1 for all the brands (due to the large size of the outside alternative), the price elasticities are in general higher for higher priced brands.

As mentioned, since the effect of sales promotions on brand equity that carries over from period to period is negative, the long-term promotional elasticity is negative for both Tropicana and Minute Maid. It is larger in magnitude for Tropicana, the brand for which the adverse effect of sales promotions on brand equity is larger and statistically significant. Interestingly, the total promotion elasticities are still positive for both brands, albeit smaller than the corresponding short-term promotional elasticities (see Table 3). Consequently, we would expect myopic versus forward-looking decision-makers to allocate a higher than optimal budget for sales promotions.

The short-term market share elasticities with respect to *advertising* for Tropicana and Minute Maid are 0.06 and 0.05 respectively. Thus, Tropicana's market share is more responsive to advertising than Minute Maid's market share. Note that the short-term advertising elasticity is much smaller in magnitude than the corresponding short-term promotional elasticity for both brands. The stronger positive effect of sales promotions in generating short-term sales may explain the increasing budget allocated to sales promotions at the expense of advertising in recent years (Jedidi et al. 1999). The long-term market share elasticity with respect to advertising is slightly larger in magnitude for each brand as compared to the corresponding short-term elasticity. This is because advertising spending has a positive impact on brand equity that carries over from period to period. We find that the brand with higher equity, Tropicana (0.32), has a much higher long-term elasticity than Minute Maid (0.21).

Although the market share elasticities with respect to advertising and sales promotions are positive, it may still not be profitable to increase their corresponding budgets. In order to assess if an increase in either the advertising or promotion budgets may be profitable either in the short-term or in the long-term, we compute the short-term and the long-term *profit elasticities* with respect to these instruments. In computing the profit elasticities, we assume that the marginal cost is 30% of the wholesale price and the market comprised of the entire Chicago market with a population of 3.16 million households.

Table 4 displays the short-term and long-term profit elasticities with respect to sales promotions and advertising. We find that the *short-term* profit elasticity with respect to *sales promotions* is positive for both brands. This implies that increasing sales promotions by 1% will be profitable for both Tropicana and Minute Maid if one considers only the short-term effects of sales promotions.[18] However, if one considers the adverse long-term impact of promotions, the total effect of sales promotions on profitability is negative for both brands. Note in this connection that we have not considered the fixed cost of promotions while computing the promotion elasticities. Inclusion of the fixed cost will, of course, decrease both the short- and long-term profit elasticities.

We observe in Table 4 that the *short-term* profit elasticities with respect to *advertising* are negative for both brands. This implies that the short-term incremental revenues generated by increasing advertising do not cover the incremental advertising costs. On the other hand, the long-term elasticities are positive since an increase in advertising in period *t* leads to an increase in brand equity and to increases in market shares in subsequent periods even if there is no increase in advertising in those periods. However, we find that the *total* profit elasticity with respect to advertising – short-term plus long-term profit elasticity -- is negative for both brands. This implies that it will not be profitable for either Tropicana or Minute Maid to increase its advertising spending. This is consistent with the findings of Lodish and his co-authors (1995) who report that generally advertising has a very small impact in mature categories and that changes in advertising expenditure have a much smaller impact than changes in advertising copy and quality. Hence, considering that the category we are studying is a mature one, we do not expect high returns to increased advertising expenses. In their analysis of another consumer packaged goods category, Jedidi, Mela, and Gupta (1999) also find that some brands do over spend on advertising and may benefit by cutting their advertising expenses. Nevertheless, the negative advertising profit elasticities should be interpreted with some caution given that we do not have the actual regional advertising data but have inferred them from the national advertising levels. Moreover, as stated earlier, the advertising data include the advertising dollars spent on all the products manufactured by these brands including frozen and refrigerated non-orange juice drinks.

6.2 Optimal Advertising and Promotion Budget Allocation: The Supply Side

[18] Note that we haven't considered the fixed cost of sales promotions.

In this section, we investigate the implications of the demand side estimates for optimal advertising and promotion levels. First, we investigate the optimal policies by myopic decision makers who consider only the current period profits. Specifically, we consider the equilibrium myopic advertising and promotion levels by setting the discount factor, δ, in Equation 15 to be equal to 0. We then discuss the equilibrium policies of forward-looking decision makers by setting $\delta = 0.95$.

Recall that we evaluate these strategy profiles as functions of the following state variables: (a) the brand equities of Tropicana and Minute Maid, and (b) seasonality as characterized by the quarterly dummies. While the brand equities are continuous state variables, the quarterly dummies are discrete. We evaluate these equilibrium policies where the dynamic components of brand equities, ζ_{jt}, range from –2 through 3. Note that this range, especially the maximum value is much greater than the range of brand equities observed in the demand side. We evaluate the equilibrium policies over a wider range of values to obtain a general intuition for the shape of these equilibrium policy functions. Hence, the equilibrium policies for the brand equity values beyond the observed range should be considered with caution. Recall that the total brand equity, $\beta_{0jt} = \overline{\beta}_j + \zeta_{jt}$. Correspondingly, the only component of brand equity that changes with time is the dynamic component, ζ_{jt}. Given the estimates of $\overline{\beta}_j$ in Table 2, it is trivial to transform the range of the dynamic component to the corresponding range of the brand equities.

6.2.1. Equilibrium Promotion Policies

We report the equilibrium myopic promotion levels for quarter 1 for Minute Maid and Tropicana in Figures 1 and 2. The equilibrium promotion levels are decreasing in own brand equity and increasing in competitor's brand equity. We observe that the equilibrium promotion policies are symmetric across the two brands with slightly higher optimal promotion levels for Tropicana. The intuition behind the shape of the equilibrium promotion function and the higher equilibrium promotion levels for Tropicana may be obtained by considering the profit function in Equation 11.[19] For the sake of convenience,

[19] The objective function in the myopic case will be a function of this profit function.

consider the case when there is no consumer heterogeneity. The first order condition with respect to the promotion level would imply that the optimal promotion level can be written as

$$\begin{aligned}\Pr^{*}_{jt} &= \rho(W_j - c_j) - \frac{s_{jt}}{\left(\partial s_{jt} / \partial \Pr_{jt}\right)} \\ &= \rho(W_j - c_j) - \frac{1}{\{(\gamma + \Gamma_j^{\Pr})(1 - s_{jt})\}}\end{aligned} \quad , \qquad (17)$$

where γ is the direct effect of promotion on the utility (in the observation equation), and $\Gamma_j^{\Pr}$ is the effect of promotion through its effect on the brand equity for brand *j*. From Equation 17, given that the overall short-term effect of promotion, ($\gamma + \Gamma_j^{\Pr}$), is positive, it is evident that the optimal myopic promotion levels are a decreasing function of the brand's market share. Since the market share is a monotonically increasing (decreasing) function of own (competitor) brand equity, the equilibrium promotion levels are decreasing (increasing) in own (competitor) brand equity. Similarly, Equation 17 implies that all else being equal, the optimal promotion levels are higher for the brand that has a higher wholesale price.[20]

We now turn to the forward-looking equilibrium promotion policies. We present the equilibrium forward-looking promotion policies for Minute Maid in Figure 3. These differ from the myopic case as follows: (i) the equilibrium promotion levels are initially increasing (decreasing) in own (competitor's) brand equity and then decreasing (increasing), (ii) they are lower than the equilibrium promotion levels under the myopic case. On the other hand, the significant adverse effect of sales promotions on Tropicana's equity implies that in case of Tropicana, zero promotions are optimal for all the levels of the state variables. The lower promotion levels can once again be attributed to the fact that forward-looking decision makers consider the negative long-term effect of promotions and hence, choose lower levels of promotions compared to myopic decision makers.

[20] The optimal promotion levels also depend on the negative effect of promotions with brands that are less adversely affected by promotions having higher optimal promotion levels. Since the detrimental effect of promotions is higher for Tropicana but not for Minute Maid, there is a tradeoff between the effect of higher wholesale prices and that of higher detrimental effect of promotions.

We now compare the equilibrium myopic and forward-looking promotion policies for the two brands with the actual promotion policies of the two brands, Minute Maid and Tropicana. In addition, we consider an intermediate scenario wherein the discount factor, $\delta = 0.5$. This value of the discount factor implies that in the current period, profits two years hence (after eight quarters) are worth only 0.4% of their value. This is akin to making decisions based on a two year time frame, a planning horizon that may be closer to current industry practice. In order to make this comparison, we compute the average promotion levels for the two brands across all the 32 stores. The equilibrium policies are computed as follows. For each period, we determine the average value of the dynamic component of brand equity, ζ_{jt}, across all the 32 stores and the quarter of the year corresponding to that period. We then compute the equilibrium advertising and promotion levels corresponding to these values of the state variables using Chebyshev interpolation (Judd 1999). We present the comparison over time for the two brands in Figures 4 and 5. We also present a comparison of the average sales promotion levels for the two brands across the four scenarios in Table 5.

The results reveal that in case of both Tropicana and Minute Maid, the forward-looking promotion levels are significantly lower than the myopic ones.[21] The optimal promotion levels for the case when $\delta = 0.5$ lie in between these two extremes for both brands. A comparison of the average values in Table 5 implies that while the optimal myopic promotion levels are higher than the actual levels for both brands, the forward-looking promotion levels are significantly lower. Further, Table 5 reveals that while the actual promotion levels are closer to the case when $\delta = 0.5$ for Minute Maid, they are closer to the myopic levels for Tropicana. Recall that in case of Tropicana zero promotion levels are optimal under the forward-looking case. The optimality of the lower forward-looking promotion levels compared to the actual values is consistent with our finding that the profit elasticity with respect to promotions is positive in the short-term but negative in the long-term for both brands. However, one should note that we did

[21] The forward-looking promotion levels for Minute Maid exhibit some fluctuations with seasonality because seasonality was one of the state variables in the computation of the equilibrium strategies.

not consider the fixed cost of promotions while obtaining the equilibrium promotion levels for the lack of availability of such data. If one were to include the fixed cost, the optimal promotion levels will be lower.

6.2.2. Equilibrium Advertising Policies

We report the equilibrium myopic advertising policies for Minute Maid and Tropicana in Figures 6 and 7. For the sake of convenience, we report these values for just one quarter – quarter 2.[22] The shape of the equilibrium advertising policy for Tropicana is similar to that of Minute Maid. Figures 6 and 7 reveal that the equilibrium advertising levels (for both brands) are initially increasing in own brand's equity level and decreasing in the competing brand's equity. The equilibrium advertising levels are more responsive to own brand equity than to the competing brand's equity. This may be attributed to the lower cross elasticities in models with the outside alternative. At very high values of own brand equity, the equilibrium advertising levels marginally decrease in own brand equity levels.[23] This shape of the equilibrium advertising levels can be explained based on the S-shaped nature of the logit response function. At lower values of the brand equity levels, we operate in the convex part of the S-shape where an increase in brand equity has higher returns at higher values of brand equity. Since our model implies that advertising affects utility through its effect on brand equity (Equations 4 and 6b), advertising has higher returns at higher values of brand equity. This is especially true given that we have an outside alternative wherein each individual product commands only a small fraction of the market, which in turn implies that we operate mostly in the convex region. However, beyond the point of inflection, the response is concave in utility (and hence, in brand equity). Hence, the equilibrium advertising levels are decreasing in own brand equities beyond this point. Further, Figures 6 and 7 reveal that the equilibrium advertising policies are symmetric for both brands. However, the higher advertising elasticity for Tropicana implies higher equilibrium advertising levels for this brand compared to Minute Maid.

[22] The shapes of the equilibrium policies did not vary with quarter.
[23] Although not reported, the equilibrium advertising levels are decreasing in own brand equity levels beyond this point of inflection.

We now turn to the forward-looking advertising policies. We present the equilibrium forward-looking advertising levels for Minute Maid in Figures 8 and 9. As in the myopic case, the shape of the equilibrium advertising policy functions is similar for the two brands. Overall, the forward-looking advertising strategy profiles are similar to the myopic strategy profiles viz., (i) increasing in own brand equity, (ii) decreasing in competitor's brand equity, (iii) Tropicana has higher equilibrium advertising levels as compared to Minute Maid. However, compared to the myopic case, the forward-looking equilibrium advertising levels are higher. This is because the forward-looking case takes into account the long-term effects of advertising. On the other hand, the myopic case considers only the contemporaneous effects of advertising.

In contrast to our approach, Dube et al. (2005) model the effect of goodwill (which is equivalent to our brand equity) on utility using a logarithmic function. Correspondingly, according to their specification, the market share is always strictly concave in goodwill (as opposed to our S-shape). Therefore, they report that the equilibrium advertising levels are decreasing in goodwill. Hence, the shape of the equilibrium advertising policy depends on the specification of the demand function. Clearly, despite being in the convex part of the logit response function, the logarithmic function for the effect of advertising in Equation 6b implies diminishing returns to increase in advertising and thus, ensures that it is not optimal for firms to increase their advertising levels infinitely.

As in the case of sales promotions, we present a comparison of the actual, myopic, two-year planning horizon, and the forward-looking advertising levels for the two brands in Figures 10 and 11 as well as in Table 6. The results reveal that while the forward-looking advertising levels are higher than the myopic and the two-year planning horizon levels for both brands, these are lower than the actual advertising levels. This implies that even if one were to consider the long-term effects of advertising, the brands are over-spending on advertising. In this connection, recall that that profit elasticities with respect to advertising are negative for both brands even after considering the long-term effects.

However, the result that the brands are over-advertising is subject to the following caveats. First, we infer the regional advertising levels by assuming that the brands allocate the advertising budgets proportional to the regional population of the Chicago area. If the allocation of the advertising budget to the Chicago area were significantly lower than that inferred by the regional population, the over-advertising claim may not hold. Second, while we have considered the long-term profits that may be derived from refrigerated orange juice, a brand may be viewing advertising as an investment in building brand equity so as to enable extension into new categories and also to act as a barrier to entry. Accounting for these effects may render higher levels of advertising optimal. Third, the demand and hence the revenues are computed for 64 oz. and 96 oz. refrigerated orange juice manufactured by the brands. However, the advertising levels are more likely to be set considering the revenues from all the products manufactured by these brands such as frozen juices, as well as non-orange refrigerated juices such as lemonade and grapefruit juice. Our analysis of the Dominick's data during this period revealed that 64 oz. and 96 oz. refrigerated orange juice accounted for about 52.9% of the total sales (including frozen and refrigerated non-orange juice drinks) for Minute Maid and 73.1% in case of Tropicana.

In order to understand the consequence of relaxing this last caveat, we tried two alternative modifications. First, under the assumption that the sales of orange juice as a proportion of total sales of (including frozen and refrigerated non-orange juice drinks) is equal to 52.9% for Minute Maid and 73.1% for Tropicana, we scale up the demand so that it includes the sales from all the products.[24] We then compute the MPE advertising and promotion strategies with this level of demand. In the second modification, we assume that the advertising expenses are apportioned between the various products in the same proportion of their sales. Using these scaled down advertising levels for 64 oz. and 96 oz. refrigerated orange juice, we re-estimate demand and also the MPE advertising and promotion levels. We present a summary of the average advertising levels from these two modifications in Table 7. Clearly, the modifications have reduced the gap between the actual and the forward-looking advertising levels. Nevertheless, despite the adjustments, the brands appear to be spending more than optimal on advertising. Hence, although we can infer that the brands are over-advertising, given the nature of our advertising data, we cannot exactly infer the extent to which they are over-advertising.

[24] This is tantamount to either increasing the market share s_{jt} or the market size M in Equation 12

In sum, the results from the supply side indicate the following: (i) under the myopic case, it will be optimal to increase the promotion budgets for both brands; (ii) however, the optimal forward-looking promotion levels are significantly lower than the actual levels for both brands; (iii) the myopic as well as the forward-looking advertising levels for both brands are lower than the actual levels. Hence, even if one were to consider the long-term effect of advertising, reducing the advertising budgets may be optimal. Overall, the forward-looking advertising and promotion levels imply that it is optimal for both brands to reduce their advertising and promotion levels.

7. CONCLUSIONS

We study the optimal levels of advertising and promotion budgets in dynamic markets with brand equity as a mediating variable. To this end, we develop and estimate a state space model based on the Kalman filter that captures the dynamics of brand equity as influenced by its key drivers – advertising and sales promotions. The model allows us to decompose the short-term impact of sales promotions into two components – a positive effect that induces consumers to switch to the promoted brand and a negative effect on brand equity that carries over from period to period. The negative long-term effect of promotions implies that the total effect of promotions, including the long-term adverse effect, is smaller in magnitude than the short-term effect of promotions. Therefore, we expect myopic versus forward-looking decision-makers to have higher sales promotion budgets.

Based on our demand model estimates, we compute the Markov Perfect Equilibrium advertising and promotion strategies for decision-makers under myopic, two-year planning horizon, and forward-looking scenarios. Our results reveal that the optimal myopic promotion levels are significantly higher than the corresponding levels for the forward-looking scenarios for both brands. We suggest that this result is a consequence of modeling brand equity as a mediating variable because it allows us to capture the differential impact of marketing actions in the short- versus in the long-term. Our results also reveal that the equilibrium forward-looking promotion levels are higher for Minute Maid, the brand for which the adverse long-term effect of promotions is lower. However, we observe that the actual promotion levels are higher for Tropicana versus Minute Maid, a result that is only consistent in the myopic case.

Further, our results reveal that the actual promotion levels for both brands are higher than the optimal budgets for the forward-looking as well as the two-year planning horizon scenarios, and that it may be profitable for both brands to reduce their promotion levels.

The equilibrium forward-looking advertising levels are higher for Tropicana, the brand that has a higher responsiveness to advertising. Further, the optimal forward-looking advertising levels are higher than the corresponding optimal levels for the myopic as well as the two-year planning horizon scenarios. However, these levels are higher than the actual advertising expenditures for both brands. Why are Minute Maid and Tropicana's actual advertising expenditures significantly higher than the optimal forward-looking advertising budgets? A possible explanation is that while our model considers the sales of the two brands only in the refrigerated orange juice category, their advertising expenditures may reflect the demand from all the products manufactured by these brands, including frozen orange juice and frozen and refrigerated varieties of other fruit juices. However, we find that these results on excessive advertising expenditures hold even after we make adjustments by scaling up demand to account for other products that these brands market as well as by scaling down advertising budgets to arrive at their projected advertising levels only for refrigerated orange juices. An alternative justification of their excessive advertising budgets is that Minute Maid and Tropicana managers view these expenditures as investments in brand equity that will help erect barriers to entry in categories that they dominate, and will facilitate the introduction of line extensions.

We conclude with a few comments about the limitations of our study and some directions for future research. First, our computation of the equilibrium advertising and promotion levels is based on assumptions regarding the size of the regional market, the advertising spending, marginal costs, and the retail pass-through. The availability of more precise information on these variables would clearly be useful. Second, our analysis of the effect of policy changes with respect to advertising and sales promotions is performed based on data from one chain in a regional market. To the extent that the Dominick's chain and the Chicago area is representative of the national population, the results from our analysis should provide insights on the tradeoffs between advertising and sales promotions. Again, it would be useful to have data that cut across several regional markets. Finally, it will be useful to investigate the optimal levels of advertising and promotion budgets across several categories so that we can make generalizations across a wide variety of product categories and brands.

In conclusion, we present a methodology for evaluating the optimal levels of advertising and sales promotion expenditures when these instruments have a long-term effect of brand equity. The modeling of brand equity as a mediating variable allows us to track the impact of changes in the firm's marketing programs, and thus to monitor the long-term health of the brand. We hope that the relative ease in implementing our approach will lead to more empirical work across a variety of product categories.

Appendix A

Steps in the Demand Estimation

We seek to estimate three sets of parameters in our estimation procedure based on GMM, (i) parameters $\Theta_1 = \{ \beta_{0s}, \overline{\beta}_j, \alpha, \beta, \gamma\}$ that affect the mean utility of brand *j* at time *t* in the observation Equation 4, (ii) parameters $\Theta_2 = \{R, \Gamma_j^A, \Gamma_j^P\}$ that capture the dynamics of brand equity in the system equation (Equation 6b), and (iii) parameters Θ_3 that capture consumer heterogeneity. The estimation proceeds as follows: First, for a given set of heterogeneity parameters, compute the mean utilities numerically using the contraction mapping algorithm proposed by Berry et al. (1995). Berry et al. (1995) prove that for a given set of heterogeneity parameters, there is a one-to-one mapping between the observed market shares and the mean utilities. Hence, the mean utilities are uniquely identified based on the market shares. Given these mean utilities, and the Kalman filter algorithm described below, we obtain the error terms, ξ_t, which is fed into the GMM objective function.

Steps in the Kalman Filter Estimation:

Step One: We begin at time *0* by choosing B_0 and Σ_0 to be our best guesses about the mean and the variance respectively of brand equity. In our empirical analysis, we lack genuine prior information and hence specify a diffuse prior by defining Σ_0 to be a large number (Harvey 1990). Thus at time 0, our knowledge of the unobserved state variable, the dynamic component of brand equity, ζ_0, is given by the following probability distribution, $\zeta_0 \sim N(\zeta_0, \Sigma_0)$.

Step Two: Let $\zeta_{t|\tau}$ denote the minimum-mean-square error estimate of the dynamic component of brand equity at time *t* given the model and all the observed data up through time τ. At any point in time *t-1*, we have observations of data from time 1 to *t-1* and we can summarize our knowledge of $\zeta_{t-1|t-1}$ as follows:

$$\zeta_{t-1|t-1} \sim N(\zeta_{t-1|t-1}, \Sigma_{t-1|t-1})$$

$\zeta_{t-1|t-1}$ is thus the posterior distribution we obtain at *t-1* after observing data *t-1*. Now our best guess for ζ_t at *t-1* i.e., $\zeta_{t|t-1}$ and $\Sigma_{t|t-1}$ is given by:

$$\zeta_{t|t-1} = R\ \zeta_{t-1|t-1} + \Gamma^{Ad} \ln(1+Ad_t) + \Gamma^{\Pr} \Pr_t \qquad \text{(A1)}$$

$$\Sigma_{t\,|\,t-1} = R'\Sigma_{t-1} R + Q,$$

where Q is a JxJ (J=number of brands) diagonal matrix with $\sigma^2_{e_j}$ as the diagonal elements. This is our prior distribution for the unobserved expectations.

Step Three: Prior to observing mean utilities at time t, our best guess for the mean utility vector is given as:

$$\delta_{t|t-1} = \overline{\beta} + R\zeta_{t-1} + \beta X_t + \alpha \Pr_t \qquad \text{(A2)}$$

Step Four: Once we recover the actual mean utility vector by "inverting" the market share in time t (i.e., δ_t) we can calculate the prediction error in our forecast and the conditional variance of this prediction error. These are used as inputs in the estimation procedure.

$$\text{Prediction Error} = \xi_{t\,|\,t-1} = \delta_t - \delta_{t\,|\,t-1} = \delta_t - \overline{\beta} + R\zeta_{t-1} + \beta X_t + \alpha \Pr_t \qquad \text{(A3)}$$

$$\text{Variance of the prediction errors} = S_{t\,|\,t-1} = \Sigma_{t\,|\,t-1} + V \qquad \text{(A4)}$$

where V is a JxJ (J=number of brands) diagonal matrix with σ^2_{ξ} as the diagonal elements.

Step Five: Given our information on δ_t and also the other variables Ad_t and Pr_t, we can update our estimate of the vector of state variables ($\zeta_{t\,|\,t}$) and the associated variance-covariance matrix ($\Sigma_{t\,|\,t}$). The exact expression for the posterior distribution of brand equity is obtained by specifying the joint normal distribution of ζ_t and forecast error ε_t conditional on observed data. The definition of conditional normal is used to obtain the optimal forecast of $\zeta_{t\,|\,t}$ conditional on observed forecast error $\varepsilon_{t\,|\,t-1}$. The exact expressions are given as below:

$$\zeta_{t\,|\,t} = \zeta_{t\,|\,t-1} + \Sigma_{t\,|\,t-1} (S_{t|\,t-1})^{-1} \varepsilon_{t\,|\,t-1} \qquad \text{(A5)}$$

$$\Sigma_{t\,|\,t} = \Sigma_{t\,|\,t-1} - \Sigma_{t\,|\,t-1} (S_{t|\,t-1})^{-1} \Sigma_{t\,|\,t-1} \qquad \text{(A6)}$$

Step Six: We use $\zeta_{t\,|\,t}$ and $\Sigma_{t\,|\,t}$ as inputs in the next round for generating prediction equations $\zeta_{t+1\,|\,t}$ and $\Sigma_{t+1\,|\,t}$ in step two. We continue the recursions till t=T the end of the sample.

Appendix B
Steps in Computing the Markov Perfect Equilibrium Advertising and Promotion Strategies

Given the demand side estimates, we have information on how advertising and promotions affect brand equity and hence both current and future profits. In computing the Markov perfect equilibrium strategies, we use brand equity as the state variable. In our case, we have two state variables – brand equities of the two brands.

1. Discretize the continuous state space (brand equities of the two brands) into finite number of grid points (20 in our case) in each dimension. The grid points can be chosen as the Chebyshev zeros with the end points of the state variables defined in the interval [a, b] and [c, d]. Please refer to Judd (1999, page 238).
2. Start with initial guesses of the strategy profile (advertising and promotion decisions for each brand at each grid point).
3. Compute the value function that satisfies the Bellman equation using contraction mapping. The integration is performed using Gauss-Hermite quadrature with seven nodes. The procedure is as follows:
    a. For each iteration within the contraction map using the last iteration value functions, interpolate the value function at the Gauss-Hermite quadrature nodes using a Chebyshev tensor polynomial of degree 4.
    b. Compute the integral with the value functions evaluated at the seven nodes using Gauss-Hermite quadrature to evaluate the value function at each grid point.
    c. Compare these value functions with those from the previous iteration. If the difference is lower than the tolerance, the contraction mapping has converged. Else, repeat steps a through c.
4. For each grid point, for each brand, compute the advertising and promotion levels that will maximize the Bellman equation. Once again, the integral is computed using Gauss-Hermite quadrature.

5. Compare these advertising and promotion policies with those in the previous iteration. If difference is smaller than the permissible tolerance, we have converged at the Markov perfect strategy profiles. If not, repeat steps 3 through 5.

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Table 1

Descriptive Statistics

BRAND	AVERAGE QUARTERLY MARKET SHARE	AVERAGE REGULAR PRICE (CENTS/OZ.)	AVERAGE REGULAR WHOLESALE PRICE (CENTS/OZ.)	AVERAGE QUARTERLY ADVERTISING ('000 $)	AVERAGE QUARTERLY PROMOTION (CENTS/OZ.)
MINUTE MAID	21.73%	3.568	2.642	125.24	0.348
TROPICANA	41.50%	3.935	2.901	191.81	0.399

Table 2

Estimates of the Demand Model Using the Kalman Filter[25]

	PARAMETER	ESTIMATE	STD ERR	T VALUE
OBSERVATION EQUATION	PRICE	-10.078	2.046	-4.925
	PROMOTION	0.965	0.247	3.904
	QUARTER 1	0.102	0.050	2.049
	QUARTER 2	-0.277	0.078	-3.559
	QUARTER 3	-0.266	0.070	-3.787
	CONSTANT (MM)	-3.817	0.523	-7.302
	CONSTANT (TROP)	-3.446	0.656	-5.251
SYSTEM EQUATION	CARRYOVER	0.773	0.081	9.534
	PROMOTION (MM)	-0.072	0.091	-0.792
	PROMOTION (TROP)	-0.109	0.066	-1.661
	ADVERTISING (MM)	0.047	0.025	1.919
	ADVERTISING (TROP)	0.063	0.034	1.881
HETEROGENEITY PARAMETERS*	SIGMA_1	0.434	0.748	0.581
	SIGMA_2	0.226	0.544	0.415
	SIGMA_PRICE	4.422	0.958	4.616
	SIGMA_PROM	0.463	0.514	0.901
	SIGMA_12	-0.666	0.735	-0.907

*The heterogeneity parameters are to be interpreted as follows
Minute Maid brand equity heterogeneity variance = $(\text{SIGMA\_1})^2$
Tropicana brand equity heterogeneity variance = $(\text{SIGMA\_2})^2 + (\text{SIGMA\_12})^2$
Minute Maid Tropcana brand equity heterogeneity covariance = (SIGMA_1)*(SIGMA_12)
Price heterogeneity variance = $(\text{SIGMA\_PRICE})^2$
Promotion heterogeneity variance = $(\text{SIGMA\_PROM})^2$

[25] Because of space considerations, we have not presented the store specific dummies.

Table 3

Implied Market Share Elasticity Estimates

	BRAND	SHORT-TERM	LONG-TERM	TOTAL
PRICE	MINUTE MAID	-1.572		-1.572
	TROPICANA	-1.625		-1.625
SALES PROMOTION	MINUTE MAID	0.275	-0.075	0.2
	TROPICANA	0.409	-0.249	0.16
ADVERTISING	MINUTE MAID	0.046	0.207	0.253
	TROPICANA	0.06	0.316	0.366

Table 4

Implied Profit Elasticity Estimates

	BRAND	SHORT-TERM	LONG-TERM	TOTAL
SALES PROMOTION	MINUTE MAID	0.0057	-0.083	-0.0773
	TROPICANA	0.0119	-0.262	-0.2501
ADVERTISING	MINUTE MAID	-2.035	0.324	-1.711
	TROPICANA	-2.143	0.584	-1.559

Table 5

COMPARISON OF THE AVERAGE SALES PROMOTION LEVELS (CENTS PER OZ.)				
BRAND	ACTUAL	MYOPIC	DELTA=0.5	FORWARD-LOOKING
MINUTE MAID	0.348	0.380	0.324	0.049
TROPICANA	0.399	0.443	0.295	0.000

Table 6

COMPARISON OF THE AVERAGE ADVERTISING LEVELS (THOUSANDS OF DOLLARS)				
BRAND	ACTUAL	MYOPIC	DELTA=0.5	FORWARD-LOOKING
MINUTE MAID	125.24	8.877	12.261	32.874
TROPICANA	191.81	15.664	21.727	60.162

Table 7

COMPARISON OF AVERAGE ACTUAL AND FORWARD-LOOKING ADVERTISING LEVELS UNDER ALTERNATIVE SPECIFICATIONS (THOUSANDS OF DOLLARS)				
	MINUTE MAID		TROPICANA	
	ACTUAL ADVERTISING	FORWARD-LOOKING ADVERTISING	ACTUAL ADVERTISING	FORWARD-LOOKING ADVERTISING
WITH SCALED UP DEMAND	125.24	95.03	191.81	98.17
WITH SCALED DOWN ADVERTISING	66.25	56.39	140.21	76.79

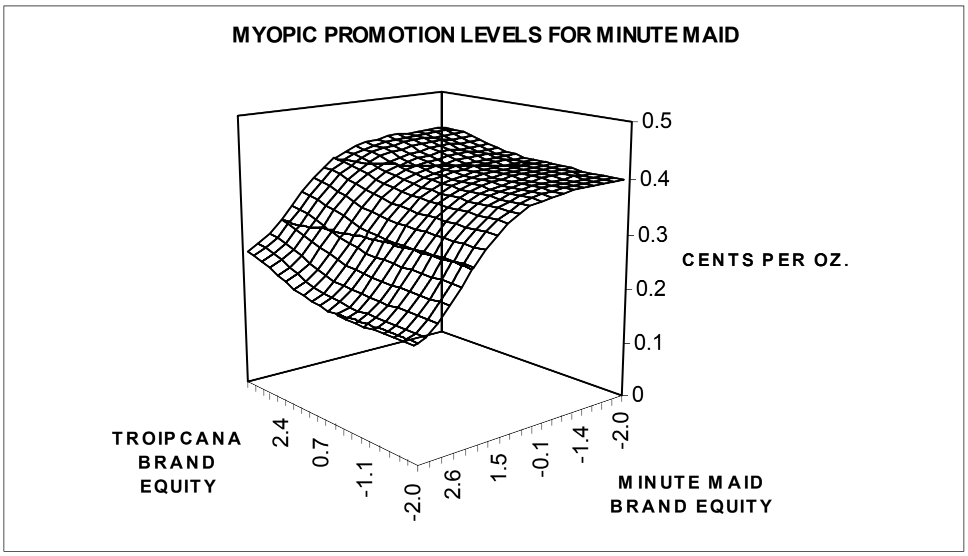

Figure 1

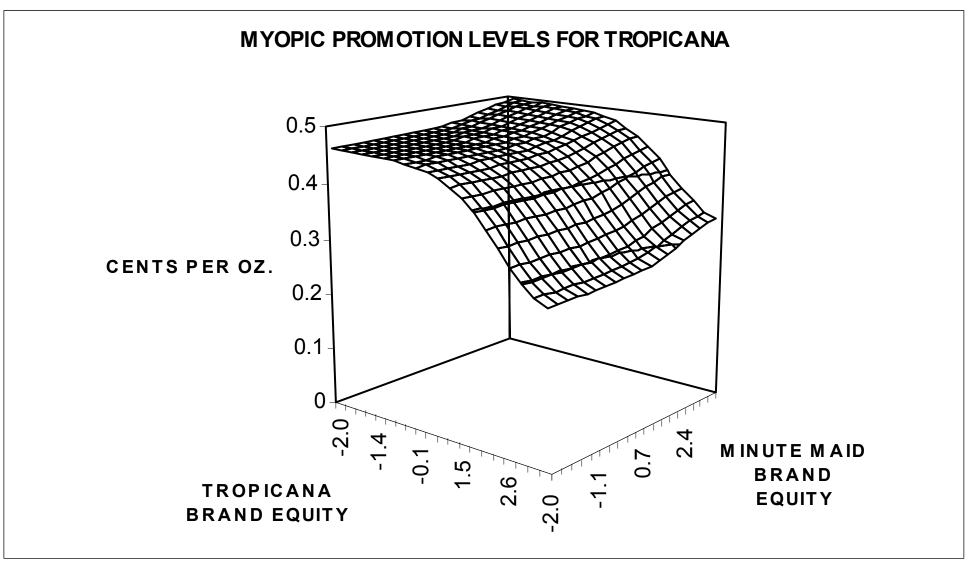

Figure 2

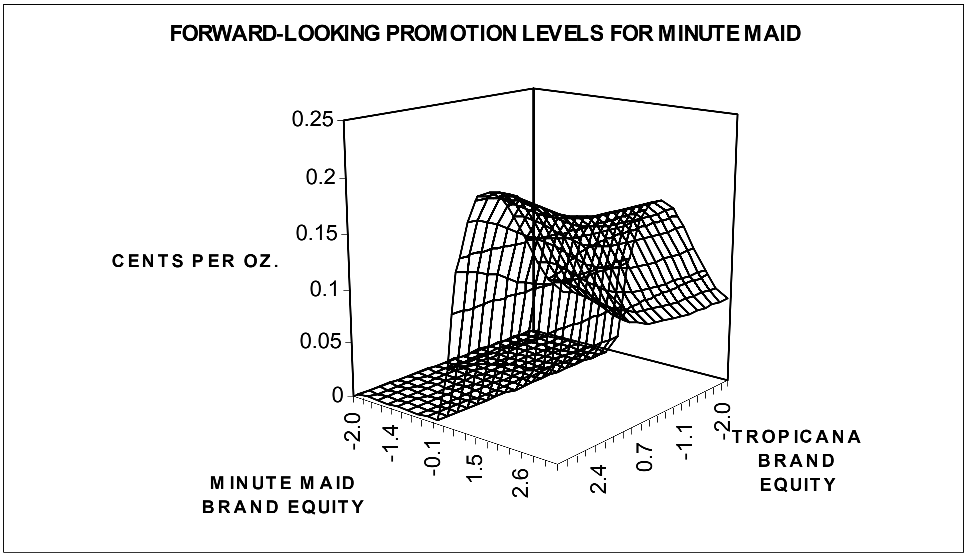

Figure 3

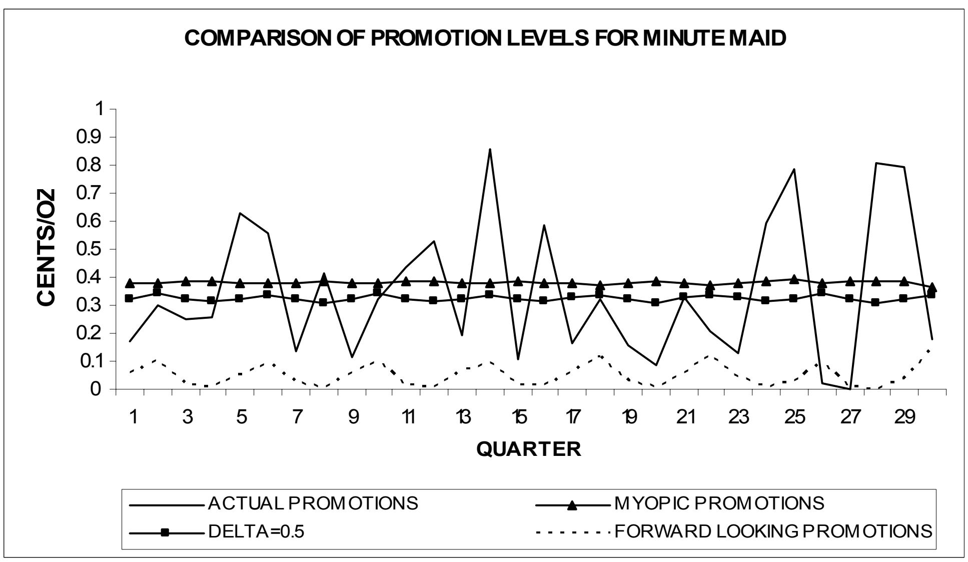

Figure 4

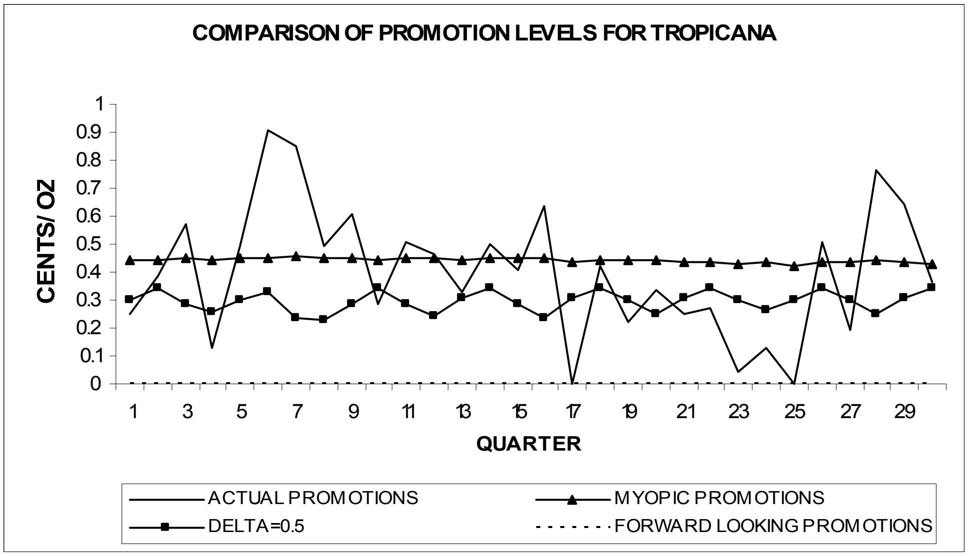

Figure 5

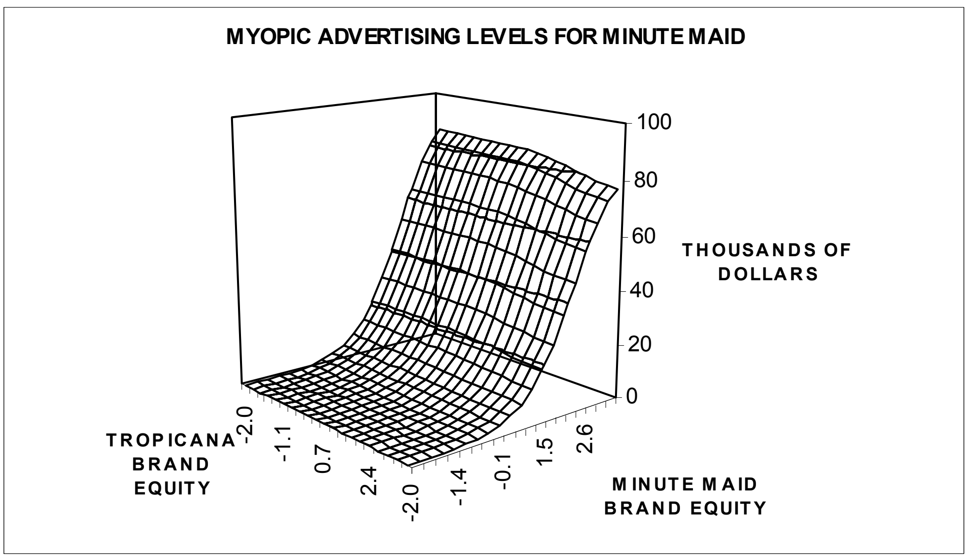

Figure 6

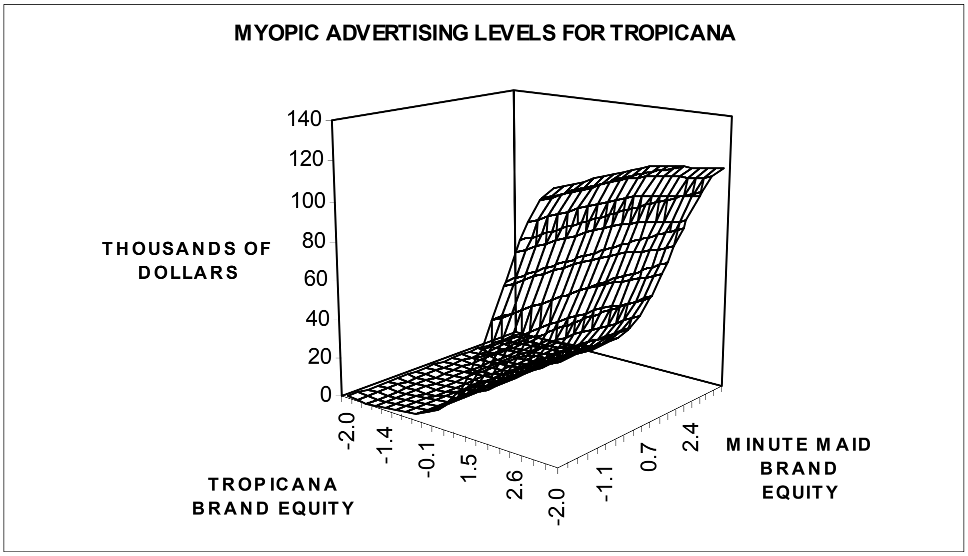

Figure 7

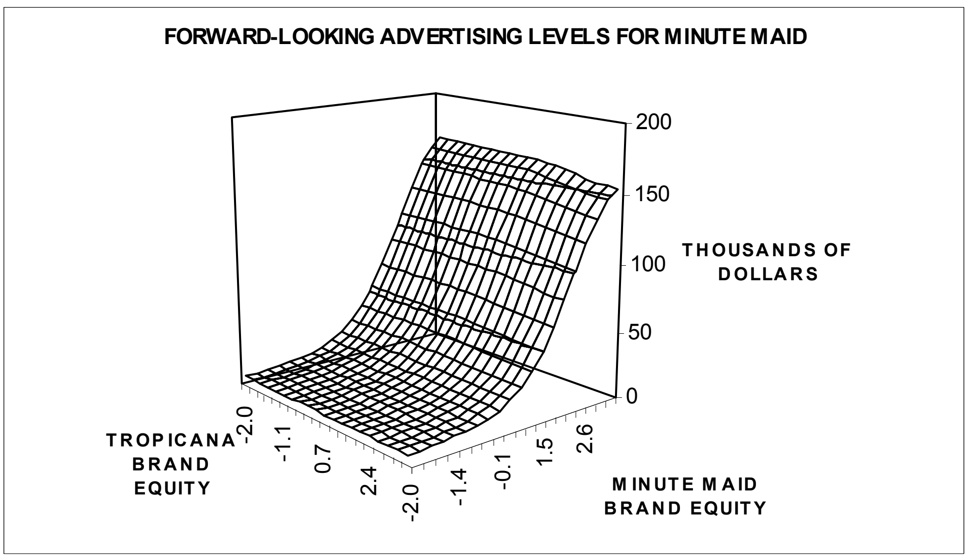

Figure 8

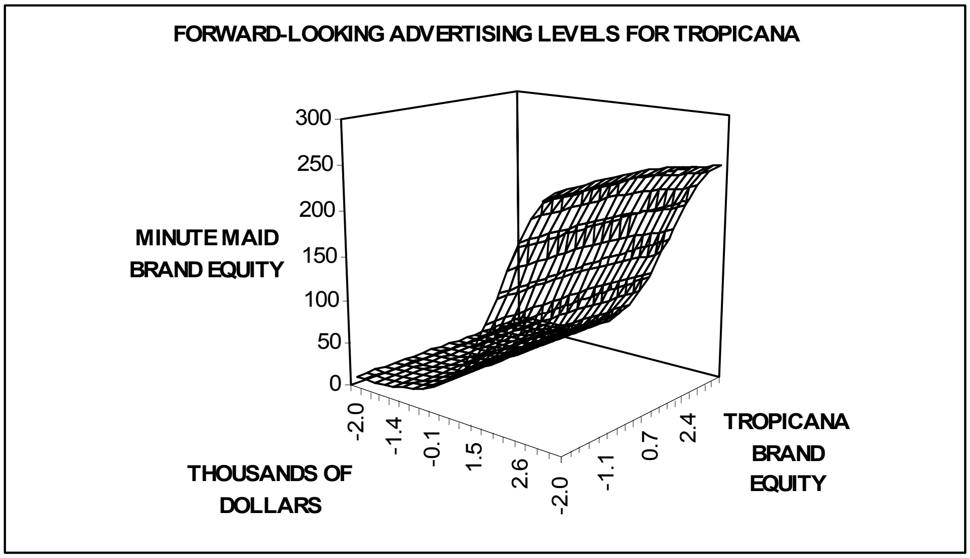

Figure 9

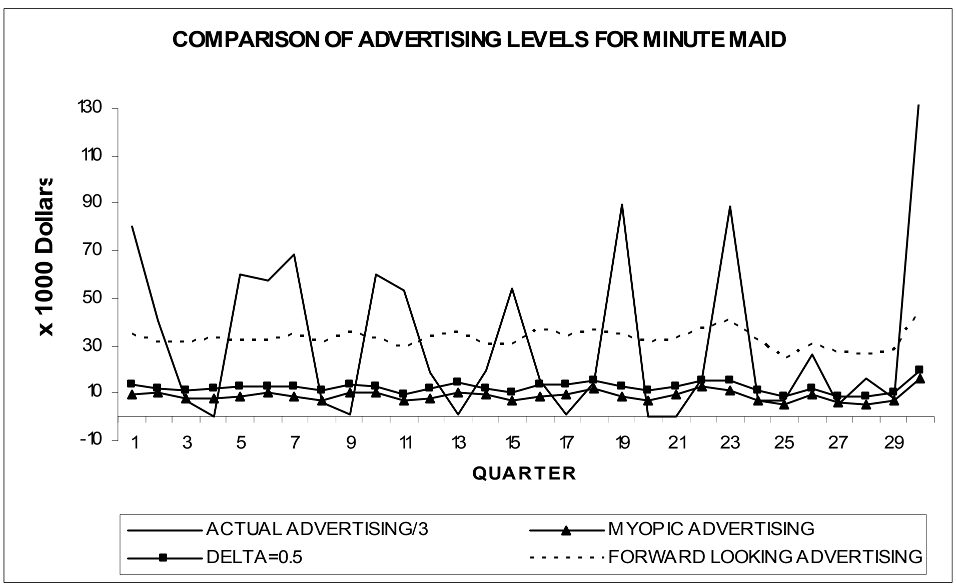

Figure 10

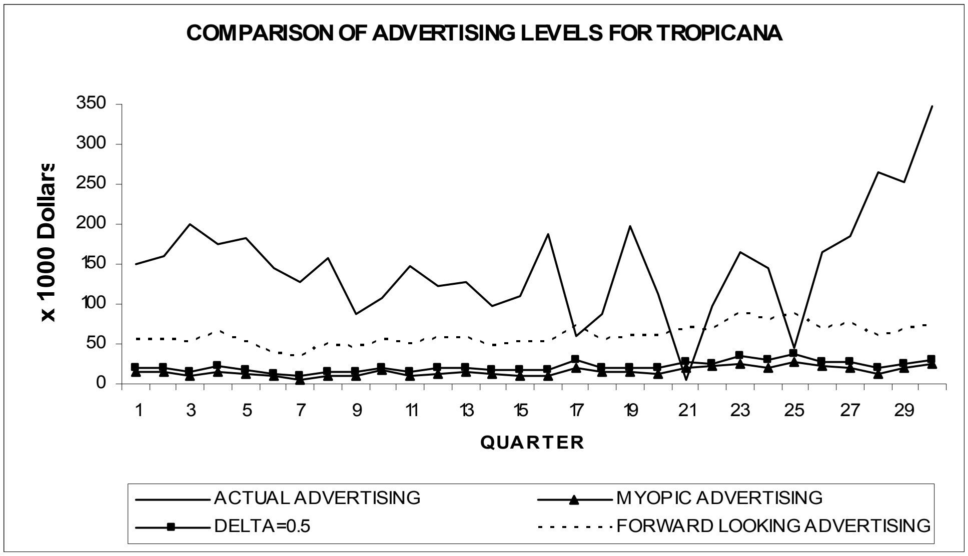
COMPARISON OF ADVERTISING LEVELS FOR TROPICANA
x 1000 Dollars
350
300
250
200
150
100
50
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
QUARTER
ACTUAL ADVERTISING
MYOPIC ADVERTISING
DELTA=0.5
FORWARD LOOKING ADVERTISING

Figure 11